

Book of Abstracts

Non standard transport
NsT@GSSI

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Large deviations for intermittent systems

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Recently a number of rigorous results has been established, concerning relationships between polynomial mixing speed for intermittent systems and large deviation properties of Birkhoff sums. We show that in many cases this turns out to represent a powerful method to investigate correlation decay for weakly chaotic systems.

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R. Artuso, C. Manein, Phys.Rev. E80, 036210 (2009)

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Anomalous diffusion: from single molecules in the live cell to cold atoms in optical lattices

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In the first lecture we will briefly discuss basic concepts from probability theory introducing simple tools. The Gauss-Lévy central limit theorem is outlined from the pedestrian point of view [1], and a brief introduction to renewal theory [2] and the continuous time random walk [3] will lead us to the mathematical description of anomalous diffusion in some strongly disordered materials [1,3] (see Chapter 1-3 in [4] for background and [5]).

In the second lecture we will show how anomalous non-stationary transport, with power law distributed trapping times, leads to the concept of weak ergodicity breaking introduced by Bouchaud in the context of glass dynamics. Here the time averages of physical observables are non identical to the corresponding ensemble averages [6, 7]. Examples from recent experiments on single molecules diffusing in the live cell, and blinking quantum dots are provided (see [8] for a popular introduction).

We then analyze strong anomalous diffusion [9], and the focus will shift to super-diffusion. We will show how basic concepts of scaling are broken using an example of motion of tracer particles in live cells which is faster than normal due to continuous supply of energy by ATP (active transport). Here, moments of the motion $\langle |x|^q \rangle \sim t^{q\nu(q)}$ with a non linear spectrum $\nu(q)$ not equal a constant. This is shown to be related to the recently introduced (in Physics) concept of infinite densities. Namely the asymptotic states are described by non-normalized densities. The infinite density is complementary to the Gauss-Lévy central limit theorem and plays an important role in the description of anomalous transport with bi-fractal spectrum [10].

Finally, and time permitting, we will discuss anomalous transport of cold atoms in optical lattices. Recent experiments showed that motion of ^{87}Rb atoms is described by Lévy laws (superdiffusion) due to a strange friction mechanism induced by the confining laser fields. Counter intuitively the friction may decrease as the particles become faster. Starting with a semi-classical description, we find [11] three phases of the motion, normal diffusion, Lévy diffusion, and $x \sim t^{3/2}$ scaling the latter related to Richardson's diffusion found in turbulence. We show how the stochastic frameworks discussed in first part of the mini-course, are derived from a microscopical model [12].

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[4] J. Klafter and I. M. Sokolov First Steps in Random Walks Oxford University Press (textbook).

[5] Course: Stochastic Processes in Physics on Eli Barkai's web page. <https://faculty.biu.ac.il/barkaie/teaching.html>

- [6] G. Bel, E. Barkai Phys. Rev. Lett. 94 240602 (2005). Y. He, S. Burov, R. Metzler, E. Barkai Phys. Rev. Lett. 101, 058101 (2008).
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- [9] P. Castiglione, A. Mazzino, P. Muratore-Ginanneschi, and A. Vulpiani, Physica D 134, 75 (1999).
- [10] A. Rebenshtok, S. Denisov, P. H nggi, and E. Barkai Phys. Rev. Letters 112,a110601 (2014). *ibid* Phys. Rev. E. 90, 062135 (2014).
- [11] D. A. Kessler, and E. Barkai Phys. Rev. Lett. 108, 230602 (2012).
- [12] E. Barkai, E. Aghion, and D. Kessler Physical Review X 4, 021036 (2014).

Relative dispersion of tracers in turbulent flow: scaling, models and irreversibility

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Pairs of tracers advected by a turbulent flow separate in an explosive manner following the celebrated Richardson law. The goal of this lecture is to introduce some phenomenological understanding of this law and to present its implications in turbulent transport. More specifically we introduce a heuristic scenario, in which pairs of tracers undergo a succession of independent ballistic separations during time intervals whose lengths fluctuate. This approach suggests that the logarithm of the distance between tracers self-averages and performs a continuous-time random walk. This leads to specific predictions for the probability distribution of separations, that differ from those obtained using scale-dependent eddy-diffusivity models (e.g. in the framework of Richardson's approach). Such predictions are tested against high-resolution direct numerical simulations and are used to discuss the question of time reversibility in turbulent transport.

S. Thalabard, G. Krstulovic, and J.Bec. "Turbulent pair dispersion as a continuous-time random walk." *Journal of Fluid Mechanics* 755 (2014): R4.

Turbulent unmixing of motile phytoplankton

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We study the spatial distribution of gyrotactic motile microorganisms transported by a three-dimensional turbulent flow generated by direct numerical simulations. We find that gyrotaxis combines with turbulent fluctuations to produce small scales clustering. We explain this result by showing that gyrotactic swimming cells behave like tracers transported by an effective compressible flow. The compressibility is explicitly derived in the limits of fluid acceleration much larger and smaller than the gravity.

F. De Lillo, M. Cencini, W.M. Durham, M. Barry, R. Stocker, E. Climent and G. Boffetta "Turbulent Fluid Acceleration Generates Clusters of Gyrotactic Microorganisms", *Phys. Rev. Lett.* 112, 044502 (2014).

W.M. Durham, E. Climent, M. Barry, F. De Lillo, G. Boffetta, M. Cencini and R. Stocker "Turbulence drives microscale patches of motile phytoplankton" *Nature Comm.* 4, 2148 (2013).

Complexity in the transport of biomolecules across nano-channel

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The transport of long biopolymers (proteins and nucleic acids) across nanoscale pores is a complex process, known as translocation, which is activated via several structural rearrangements. Generally, the basic chemical and physical properties of the molecules strongly determine modes and rates at which their transport in nanopores occurs; this principle constitutes the core for the analysis and sequencing of biomolecules by nanopore devices.

In this perspective, the theoretical knowledge of microscopic mechanisms influencing the translocation dynamics becomes crucial not only to understand the transport at a cellular level but also to improve the various biotechnology applications.

We show that a satisfactory description of the complex phenomenology observed in experiments on biopolymer translocation requires an integrated approach able to combine molecular dynamics simulations, statistical analysis of the dynamics and techniques from stochastic processes theory.

Anomalous random walk and nonlinear fractional PDE's: applications in physics and biology

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Linear fractional equation involving a Riemann-Liouville derivatives is the standard model for the description of anomalous subdiffusive and superdiffusive transport of particles. The question arises as to how to extend this fractional PDE for the nonlinear case involving particles interactions. The talk will be concerned with the nonlinear fractional PDE's and aggregation phenomenon.

References related to the talk can be found here <http://www.maths.manchester.ac.uk/~sf/anomalousdiffusion/index.html>

Diffusion approximation and multi-level selection

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In the beginning of the 1900s, population genetics viewed the gene as a black box that could exist in a variety of states that were named alleles. Rare mutations that occurred at similar rates were responsible for the transitions between alleles. This conceptualization, known as beanbag genetics, was put in a firm mathematical basis by the so-called holy trinity of evolutionary population genetics: Ronald Fisher (1890-1962), Sewall Wright (1889-1988) and J.B.S. Haldane (1892-1964).

A more modern perspective considers a gene as a particular strand of DNA that typically comprises some 500 nucleotide sites (e.g., the gene coding for the mammalian hemoglobin consists of 423 nucleotide sites). As a result, the number of alleles (on the order of 4^{500}) is so much greater than typical effective population sizes (on the order of 10^5) that one can safely assume that any new mutant represents a new allele and that it is likely to be represented only once at the moment it appears in the population. These considerations imply that stochastic treatments are essential to study the time-dependence of the frequency x of the mutant in the population. This treatment was developed by Motoo Kimura (1924-1994) in the 1950s and is regarded as one of the most complete and elegant theories in all of biology. In the 2000s, Kimuras theory transcended Genetics and found application in Ecology, revolutionizing our understanding of biodiversity and biogeography.

In the first part of these lectures I plan to review Kimuras theory for the behavior of a mutant allele in a large but finite population of size N . Since the evolutionary processes of selection, mutation and migration are such that the third and higher order jump moments of x are on the order of $1/N^\alpha$ with $\alpha \geq 2$, the master equation can be approximated by a Fokker-Planck equation, or a (generalized) diffusion equation, when only terms of order $1/N$ are retained. Hence the name diffusion approximation for Kimuras stochastic treatment of the evolution of new mutants. I will focus on the calculation of the probability of fixation as well as on the length of time required for this event to occur. The relevance of these findings to the important concept of molecular clock will be addressed as well.

In the second part, I show how Kimura himself extended his diffusion equation in the 1980s to describe a population subdivided into groups that compete among themselves. The survival or reproduction efficiency of a group depends on the number of mutants it contains so the model describes a genuine two-level selection situation. This group selection model bears on the controversial issue of the evolution of altruism in the case the mutant has a selective disadvantage with respect to the other allele types but the odds of group survival increases with the frequency of mutants in the group. I will discuss the solution of the resulting non-local Fokker-Planck equation for a variety of group survival functions and its relation with the celebrated Hamiltons rule of inclusive fitness theory.

Non-Markovian Reaction-Transport: modelling biological invasion

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A common key feature of many biological invasions is the existence of rest phases during which individuals reproduce. Since this process alternates with phases of movement it is necessary to make use of a general description where both processes are coupled, i.e., cannot occur separately. However, some recent studies propose reaction-diffusion equations that lead to behaviors that cannot be physically accepted. In this talk I will show the reasons why these equations cannot be used and will present mesoscopic models adequate for biological invasions modeling.

Diffusion of small ligands in the presence of many sinks: the diffusion interaction

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The rate constant that describes the diffusive encounter/reaction between a particle and a large sphere can be computed easily by solving the stationary diffusion (i.e. Laplace) equation for the particle density with appropriate boundary conditions imposed on the surface of the sphere. In one classic, text-book example, this calculation is used to estimate the binding rate constant of a ligand to a receptor-covered cell.

But what happens if the particles are diffusing in the presence of many spheres that compete for the same ligands? In spite of the apparent complexity, the same mathematical framework as the two-body problem can be used to solve the N-body problem exactly, by resorting to addition theorems for the appropriate fundamental solutions of the Laplace equation.

I will start by illustrating in detail the solution of the three-body problem, i.e. diffusion of a ligand in the presence of two reactive spheres. A new kind of effective interaction arises between the two spheres that compete for the same ligand, known as the diffusive interaction. As a result, the overall binding rate constant for any finite separation between the two spheres is always smaller than twice the rate constant for an isolated sphere. The diffusive interaction is long-range and decays as Coulomb and gravitational energies.

I will then illustrate the solution of the general N-sphere problem along with some examples. This powerful theory can be used to model a variety of physical situations, from diffusion in complex, confining media to binding of a ligand to large, complex biomolecules with multiple binding sites.

Stochastic reaction-diffusion systems and population dynamics

SIMONE PIGOLOTTI

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I will present a stochastic reaction-diffusion model describing competition between two different biological species and study how different physical effects can determine the fittest one. An interesting problem arises when the two species diffuse at different speeds. In this case, the fastest species tends to out-compete the other thanks to a noise-induced effect that will be characterized mathematically. I will also briefly discuss the relevance of fluid flows for population dynamics.

S. Pigolotti and R. Benzi "Selective advantage of diffusing faster." *Physical Review Letters* 112.18 (2014): 188102.

Dynamics of an intruder in a granular fluid: from dilute to dense experiments

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A single experimental setup is the occasion for a tour into many issues in non-equilibrium statistical mechanics. In the experiment, a rotating intruder performs a Brownian-like dynamics under the influence of collisions with a shaken granular media. Dissipations in the form of inelastic collisions and dry tangential friction make the system inherently out of equilibrium. When the granular medium is diluted, the results agree with a Boltzmann-Lorentz description which, in the large mass limit, is well approximated by a Fokker-Planck equation for a Ornstein-Uhlenbeck process with Coulomb friction. When the intruder is shaped so as to break the symmetry under rotation-inversion, an average drift ("ratchet" or "motor" effect) is observed, with properties depending on the dominant dissipation: friction or inelastic collisions. When the density of the surrounding medium increases, non-Markovian effects appear. The first consequence is a violation of the Einstein relation which - near equilibrium - describes the linear response to a small force provided by an external motor. The analysis of a generalized fluctuation-dissipation relation explains the nature of the violation: a joint effect of dissipation and coupling with the dense fluid. When the density is increased further and the jamming transition is approached, anomalous diffusion appears in the form of transient cage effects (subdiffusion) and - more surprisingly - superdiffusion at large times.

A. Puglisi "Transport and Fluctuations in Granular Fluids" Springer 2015

A. Puglisi, A. Sarracino, G. Gradenigo and D. Villamaina (2012). Dynamics of a massive intruder in a homogeneously driven granular fluid. *Granular matter*, 14(2), 235-238

C. Scalliet, A. Gnoli, A. Puglisi and A. Vulpiani (2015). Cages and anomalous diffusion in vibrated dense granular media. *Physical Review Letters*, 114(19), 198001.